

Growth in a Restricted Solid on Solid Model with Correlated Noise

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Received August 21, 1989; final March 19, 1990

We introduce correlated growth into a restricted solid on solid model (RSOS), a stochastic deposition model with a constraint on neighboring height differences. A two-dimensional lattice model is used in which particles are deposited via horizontal Levy flight steps with a step length distribution exponent f . Though RSOS is in the same universality class as ballistic deposition for uncorrelated deposition, it appears to depart from it for strong correlations. For $f=1$, the short-range limit is reached and both exponents β and χ , which describe the dependence of surface width on time and strip length, tend to 1. For $f>1$ we retreat to an enhanced algorithm, searching for growth sites which become excessively rare. We find an unusual short-time dependence, but χ still tends to 1. The number of growth sites G shows saturation for $f<1$, while for $f\geq 1$ we observe $G/L\rightarrow 0$ as the strip length L increases. Finally, we test directly the relationship of noise-noise correlation strength to f , and find that a direct comparison between correlated growth models and theoretical predictions for growth with correlated noise is so far unjustified.

KEY WORDS: Growth; deposition; noise; correlation; roughness; stochastic.

1. CORRELATED DEPOSITION MODELS FOR ROUGH SURFACES

The idea of spatially correlated deposition for cluster growth models was introduced in studies of the fractal dimension of growth perimeters.⁽¹⁾ "Butterfly" deposition was proposed with deposition sites chosen according to a restricted Levy flight probability distribution

$$P(r) \sim 1/r^{f+1} \quad (1)$$

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where r is the distance from a current deposition site to the next and f is a step length distribution exponent. Thus, the rough surface becomes correlated unto itself, which may correspond to the existence of charged impurities. Since the mean step length depends on f , we can recognize the following patterns of deposition. For $f = -1$ the deposition is uncorrelated, according to a chosen growth model. For $0 < f < 1$, in the long-range limit, the next deposition still may occur very far away from a current site. For $f > 1$, in the short-range limit, the particles tend to be deposited as close as possible. It is clear that the surface roughness could be tuned by the parameter f . The dependence of critical exponents characterizing surface roughness on the parameter f is the focus of this paper. The importance of understanding the deposition processes, e.g., vapor deposition and electron-beam evaporation, is well recognized in materials research.⁽²⁾ The studies of growth models with correlated noise which are easy to simulate may help gain insight into the nature of noise inherent in deposition processes.

The “butterfly” deposition algorithm becomes even more transparent when realized on a flat strip, leading to columnar growth. Two possible cases of surface roughening are clearly distinguished. The simplest case is the deposition according to a horizontal Levy flight only, the distance r between columns being measured horizontally. Spatially correlated noise should result if all columns can grow with no restriction, as is the case with ballistic deposition.⁽³⁾ With somewhat restricted deposition, i.e., when not all columns possess a growth site at a given time, the distinction between spatial and temporal noise correlations is not quite obvious. Such is a case considered in this paper, where correlated RSOS model is studied.

The second case is the literal realization of the “butterfly” model on the strip, when height differences play as much a role as distances between the columns. In this case the temporal noise correlations are bound to be felt. A variation of this model is possible, of course, in which only height differences contribute to the “butterfly” probability. We expect that the second case should exhibit a different roughness dynamics and thus belong to a different universality class than the first.⁽⁴⁾ One would also expect that the restricted deposition falls into a second case or at least exhibits some features different from the first case. In fact, this is what we have found numerically, and this is the main result of our paper.

Rough surfaces are characterized⁽⁵⁾ by the surface width ω ,

$$\omega = \langle (h) - \langle h \rangle^2 \rangle^{1/2} \quad (2)$$

where h is height and $\langle \dots \rangle$ denotes averaging over the whole surface.

Exponents χ and z describe the dependence of the surface width ω on strip length L and time t (average height $\langle h \rangle$), according to a scaling law⁽⁶⁾

$$\omega(t, L) = L^\chi f(t/L^z) \quad (3)$$

where the scaling function $f(x)$ is defined by

$$f(x) \sim \begin{cases} x^\beta, & x \ll 1 \\ \text{const}, & x \gg 1 \end{cases} \quad (4)$$

and $\beta = \chi/z$.

Thus, two well-understood limits are observed for uncorrelated noise: at $t \ll L^z$, $\omega \sim t^\beta$ (short times) and at $t \gg L^z$, $\omega \sim L^\chi$ (saturation).

Three models of deposition with random noise were recently found to belong to the same universality class with exponents $\beta = 1/3$ and $\chi = 1/2$ in two dimensions, in accordance with the theoretical model of Kardar *et al.*,⁽⁷⁾ based on the nonlinear Langevin equation

$$\partial h / \partial t = v \nabla^2 h + \lambda / 2 (\nabla h)^2 + \eta(x, t) \quad (5)$$

where v , λ are parameters and $\eta(x, t)$ is random noise. These three models are the Eden model,⁽⁸⁾ ballistic deposition,⁽⁹⁾ and the recent RSOS model of Kim and Kosterlitz.⁽¹⁰⁾ While the first two models have notoriously long-lived transients, the latter model turns out to be exceptionally well converging, with excellent results even for short strips. In fact, ref. 3 quotes $\beta = 0.308 \pm 0.011$ for uncorrelated ballistic deposition based on sizes up to $L = 10^6$, while we were able to get $\beta = 0.330 \pm 0.001$ for $L = 1000$ in the RSOS model. Thus, we have chosen RSOS to study the effects of noise correlations on surface roughness, hoping to be able to obtain reliable results for reasonable system sizes. In addition, as explained above, we expected to find some new features for correlated noise, RSOS being a restricted deposition model. The model consists of random deposition with a restriction that the neighboring columns should differ in height by no more than n (we take $n = 1$ everywhere). In the case of correlated deposition in the RSOS model, for f sufficiently large, a deposition will occur next to a previous one. In this case, the height constraint becomes very relevant by restricting the configuration to have hills of maximum slope, with no flat sections. This is a special feature of the RSOS correlated deposition model, and is not present in other deposition models.

Recently, a theoretical prediction was made by Medina *et al.*⁽¹¹⁾ for the dependence of the exponents χ and z on the strength of noise correlations. For spatial noise correlations in two dimensions

$$\langle \eta(x, t) \eta(x', t') \rangle \sim |x - x'|^{2\rho - 1} \delta(t - t') \quad (6)$$

it was found that for $\rho \leq 1/4$, the exponents $z = 3/2$ and $\chi = 1/2$ remain unchanged (same as for random noise), while for $1/4 < \rho < 1$ both exponents depend on ρ linearly, reaching 1 for $\rho = 1$. Notably, the prediction implies in two dimensions that at short times the surface is smoother than in the case of random deposition ($\beta < 1/2$) for $\rho < 1/2$, and that the surface is rougher than in the case of random deposition ($\beta > 1/2$) for $\rho > 1/2$. In addition, according to ref. 11, for $0 < \rho \leq 1$ the exponents must obey $\chi + z = 2$ due to Galilean invariance, as they do for random noise. For $\chi > 1$ the prediction is not applicable, since higher-order nonlinearities become relevant.

The prediction of ref. 11 has been confirmed for ballistic deposition by Meakin and Jullien.⁽³⁾ The spatial correlations were introduced via horizontal Levy flight (first case). Apart from the fact that the relation $\chi + z = 2$ is not obeyed for $\rho > 1/2$, the agreement is quite good. However, there is no direct way to compare the parameter f to ρ . To this end, Meakin and Jullien conjectured the identity

$$f = 2\rho \quad (7)$$

based on the concept of fractal codimension. For consistency, we followed ref. 3 in comparing our results to theory⁽¹¹⁾ via $f = 2\rho$. However, in this paper we attempt for the first time to verify numerically this identity, which forms the basis of a numerical comparison between the prediction⁽¹¹⁾ and any simulation using correlated deposition (1). Our analysis shows that such direct comparison is so far unjustified.

In Section 2 we present two new growth models for surfaces with tunable roughness. In Section 3 we present the results, compared with the prediction⁽¹¹⁾ and with the simulation of Meakin and Jullien.⁽³⁾ In Section 4 we investigate the conjecture (7). Our conclusions are presented in Section 5.

2. RSOS DEPOSITION MODELS WITH CORRELATED NOISE

The first model we use (further referred to as Model 1) is based on the original Kim and Kosterlitz algorithm⁽¹⁰⁾ with correlated noise introduced via horizontal Levy flight, following Meakin and Jullien.⁽³⁾ Namely, we choose a step length δx , using a random number uniformly distributed over the range $0 < R < 1$, from

$$\delta x = R^{-1/f} \quad (8)$$

The sign of δx is positive or negative with equal probability. We truncate δx to an integer, and periodic boundary conditions are used to restrict

the column in which the deposition is made to within the strip of length L . Thus, the distribution of step lengths obeys (1). Then we check if the deposition is possible according to ref. 10, namely if the height differences between the neighboring columns do not exceed 1. If so, the particle is deposited; otherwise, we choose another step length δx and check again. Note that since we have to deal with a large number of rejections, the growth dynamics appears to be different from ballistic deposition (all columns have one growth site) and from Eden deposition (all columns have at least one growth site). In fact, the number of growth sites G in the RSOS model decreases dramatically with time from its initial value $G = L$, and more so for strongly correlated noise ($f \geq 1$). Due to this fact, the computer time becomes prohibitive, mostly being lost in searching for growth sites.

Thus, we introduced an enhanced algorithm (further referred to as Model 2) in order to facilitate finding the growth sites for $f \geq 1$. In Model 2 the step length is chosen as in Model 1. Then, we either deposit the particle if the height restriction condition is fulfilled, or we search for the closest growth site and deposit it there. Note that Model 2 is different from restructuring model,⁽³⁾ since the nearest growth site may be higher as well as lower than the current growth site. It is not clear, however, whether the probability distribution of step lengths (1) is obeyed in Model 2. The proper way to choose growth sites according to (1), as described in ref. 1, requires optimizing computer time and will be reported elsewhere.⁽¹³⁾ We hoped, however, that for strongly correlated noise the Model 2 would exhibit the same tendency as Model 1.

3. RESULTS AND DISCUSSION

We present the results of our simulations in two dimensions for Model 1 first. While the growing surface profile for the RSOS model with uncorrelated noise consists of undulating hills (see Fig. 1), the profile for strongly correlated noise is quite different. It consists, even for relatively short times, of almost perfect mountains (remining one of a ziggurat), the height and width of which are of the order of strip length L (see Fig. 2). Such a visual appearance is characteristic for all $f \geq 1$. Not surprisingly, one finds it almost impossible to proceed with simulations for $f > 1$: the growth sites become excessively rare and far from each other, while the chosen step length tends to be of the order 1.

Thus, we present our results for the Model 1 only for $f \leq 1$ (see Tables I and II). We compare our results directly to those of ballistic deposition.⁽³⁾ We follow the comparison made by ref. 3 between theoretical and numerical results via $f = 2\rho$ (but see Section 4). We used moderate



Fig. 1. Surface profile for RSOS at $f = -1$; $L = 200$.

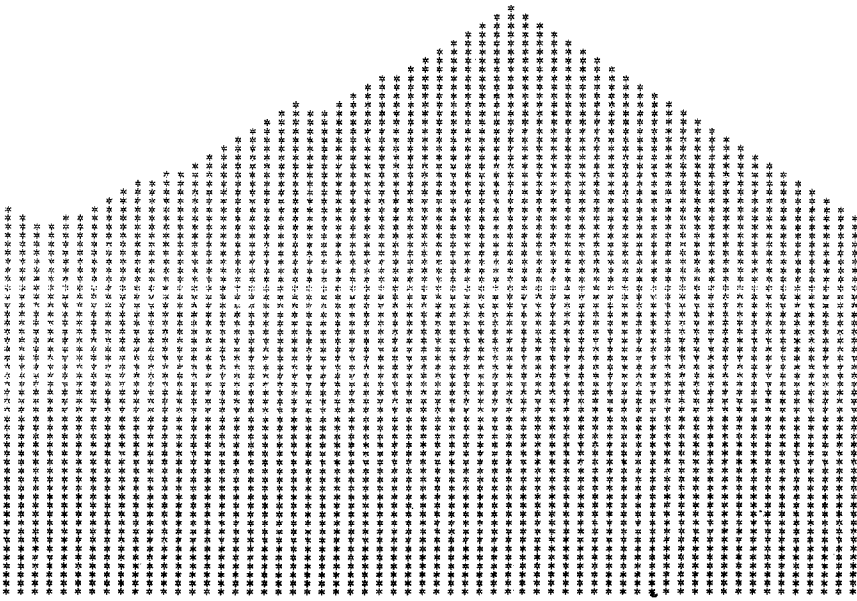


Fig. 2. Part of surface profile for RSOS at $f = 1$; $L = 1000$.

Table I. Values Obtained for β : Comparison with Theoretical Predictions and Ballistic Deposition

Correlation exponent f	$\rho = f/2$	Predicted value ⁽¹¹⁾	Ballistic deposition ⁽³⁾	Model 1
Uncorrelated				
0	0	1/3	0.308 ± (0.011)	0.330 ± (0.001)
1/4	1/8	1/3	0.347 ± (0.001)	0.41 ± (0.02)
1/2	1/4	1/3	0.370 ± (0.001)	0.46 ± (0.03)
2/3	1/3	0.381...	—	0.54 ± (0.04)
3/4	3/8	0.411...	0.430 ± (0.001)	0.61 ± (0.03)
1	1/2	1/2	0.491 ± (0.003)	1.00 ± (0.05) ⁽¹³⁾

strip lengths, typically up to $L = 2000$, averaged over 200–500 runs. The calculation of β is based on time up to $0.7L$, at which point saturation sets in. The calculations of χ are based on time up to $20L$. Normally, a single run of such length is enough to determine the average width at saturation.

β appears to be strongly dependent on L for stronger correlations ($f > 2/3$).⁽¹²⁾ The values in Table I are extrapolated from various system sizes for $1/L \rightarrow 0$. Our error ranges are due to finite-size effects, and are much larger than those from linear least-square fits. Figure 3 shows the dependence of surface width ω on time (average height) for chosen values of f . Note a slight upward curvature, more pronounced for $f = 1$, which is opposite to finite-size effects (saturation). This tendency will reappear dramatically in Model 2. In fact, for all $f > 1/4$ there appears to exist a regime of random deposition for very short times, while for $f \leq 1/4$ there is none. Thus, we quote the results of the fits excluding the random deposition regime and the saturation regime. The remaining part is quite sufficient for a good fit, and the results are convincing, since they are opposite

Table II. Values Obtained for χ : Comparison with Theoretical Predictions and Ballistic Deposition

Correlation exponent f	$\rho = f/2$	Predicted value ⁽¹¹⁾	Ballistic deposition ⁽³⁾	Model 1
Uncorrelated				
0	0	1/2	0.475	0.51 ± (0.01)
1/4	1/8	1/2	0.511 ± (0.004)	0.58 ± (0.05)
1/2	1/4	1/2	0.530 ± (0.005)	0.65 ± (0.10)
2/3	1/3	0.552	—	0.73 ± (0.15)
3/4	3/8	0.583...	0.615 ± (0.009)	0.85 ± (0.20)
1	1/2	0.666...	0.652 ± (0.005)	1.003 ± (0.002) ⁽¹³⁾

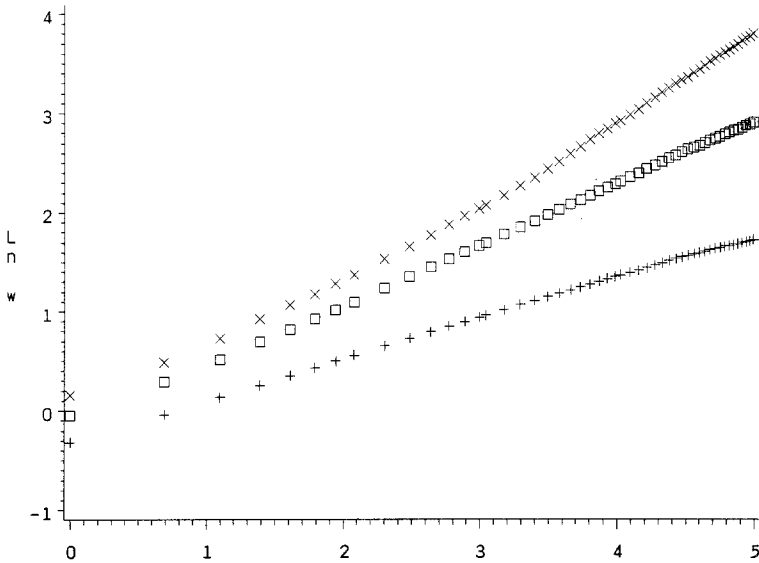


Fig. 3. Log-log plot of ω vs. t for $L=1000$: (+) $f=1/4$; (\square) $f=3/4$; (\times) $f=1$ (Model 1).

to the tendency of finite-size effects to decrease the apparent short-time exponent β .

By comparing our results to the predicted values⁽¹¹⁾ via $f=2\rho$ and to the ballistic deposition simulations,⁽³⁾ we conclude that there is obviously a disagreement for strongly correlated noise. Our results are not in agreement with the prediction⁽¹¹⁾ even for weakly correlated noise ($f < 1/2$), though we get a good estimate of RSOS for uncorrelated noise. The disturbing possibility that χ may exceed 1 for $f=1$ may be discarded in view of our new large-scale simulations.⁽¹³⁾ In fact, our preliminary results based on sizes up to $L=10,000$ confirm the results of this paper, and indicate that $\chi=1$ and $\beta=1$ at $f=1$.⁽¹³⁾ Altogether, we have established the departure of Model 1 from both theoretical prediction⁽¹¹⁾ and numerical simulations.⁽³⁾ There may be a number of reasons for correlated RSOS deposition to belong to a new universality class:

1. Model 1, being a restricted deposition model, may not correspond to spatially correlated noise models (first case): temporal noise correlations may become important.
2. The higher nonlinear terms may be called for at $f=1$: the height constraint is similar to introducing an additional operator which becomes relevant at $f \geq 1$.
3. The comparison based on $f=2\rho$ may not be valid in general, or,

in particular, for RSOS; thus, the comparison to the theoretical prediction⁽¹¹⁾ may not be valid;

4. The catastrophic loss of growth sites in case of strongly correlated noise may lead to a deterministic, rather than noisy behavior.

We will try to address all these possibilities in what follows. We note that the larger system sizes may not lead to significantly smaller exponents, since we found that even in the case of correlated random deposition, β may appear considerably larger than $1/2$ for $f > 1$, but converges to its expected value after a long time t (of the order of L). Since saturation in RSOS typically occurs before that, we might find the same effect of increased apparent short-time exponents considering larger system sizes. Our preliminary results⁽¹³⁾ confirm this observation.

The loss of growth sites increases rapidly with f (see Fig. 4). While for $f \leq 3/4$ we observe the saturation of the number of growth sites G , which occurs almost instantaneously and indicates that $G/L \rightarrow \text{const}$ for $L \rightarrow \infty$, for $f = 1$ we observe a catastrophic loss of growth sites, so that we suggest that $G/L \rightarrow 0$ for $L \rightarrow \infty$. The constant for $f = -1$ is about $1/2$, for $f = 3/4$ about $1/5$, but for $f = 1$, G typically is of order 10 for all system sizes.

We present now our results for Model 2 ($f \geq 1$). Though visually the surface profiles are indistinguishable from those of Model 1, the results for

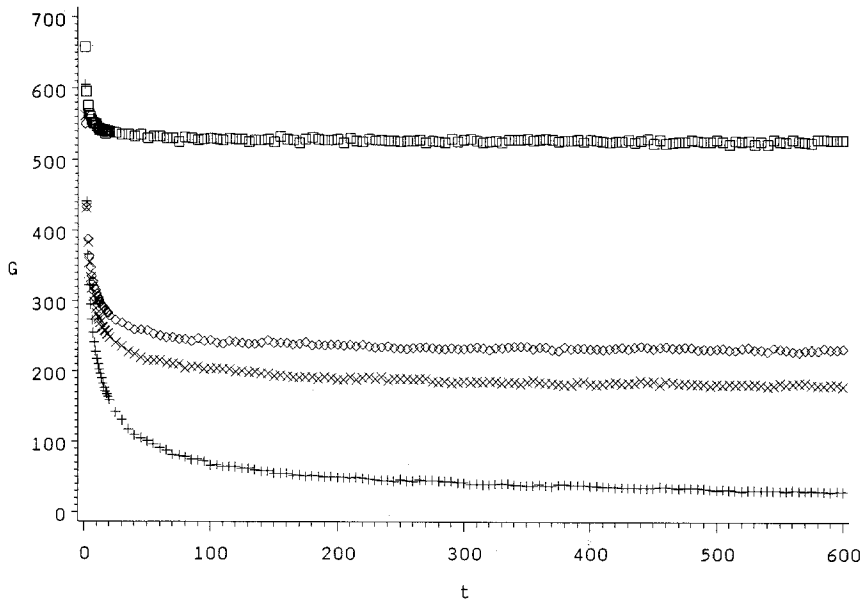


Fig. 4. Dependence of number of growth sites G on time t for $L = 1000$: (\square) $f = -1$; (\diamond) $f = 2/3$; (\times) $f = 3/4$; ($+$) $f = 1$ (Model 1).

β are qualitatively different and quite unusual. For $1 \leq f \leq 2$ we get χ of about 1, $\chi = 1.05 \pm 0.05$. There is much less fluctuation in the saturation regime for $f \geq 1$, caused possibly by the fact that the growth sites become excessively rare. Moreover, we find that the number of growth sites G saturates at about the same time as the surface width. The saturation value of G does not seem to depend much on L or f , and is typically of order 10. This prompts us to conclude that G/L tends to 0 as $L \rightarrow \infty$ for $f \geq 1$. This feature distinguishes the RSOS model from other correlated deposition models.

We had to conclude that the dependence of the surface width on time for $t \ll L^z$ does not obey a single scaling law. Our log-log plots exhibit persistent upward curvature up to the saturation regime. Roughly speaking, one can distinguish two regimes: for very short times $\beta \approx 1/2$ and for much longer times, just before saturation, $\beta \approx 1$ (see Fig. 5). These regimes are more pronounced for $f=1$ than for $f > 1$. The upward curvature was indicated for $f=1$ in Model 1; thus, the algorithm of Model 2 seems only to enhance the same tendency for $f \geq 1$. As we noted above, Model 2 was designed to enhance the growth dynamics of Model 1 for strongly correlated noise ($f \geq 1$). For $f \geq 1$ the saturation occurs at the same time as in Model 1, and the saturation widths for $f=1$ are comparable to those of Model 1. In summary, Model 2 exhibits a tendency for $\beta = 1$ and $\chi = 1$

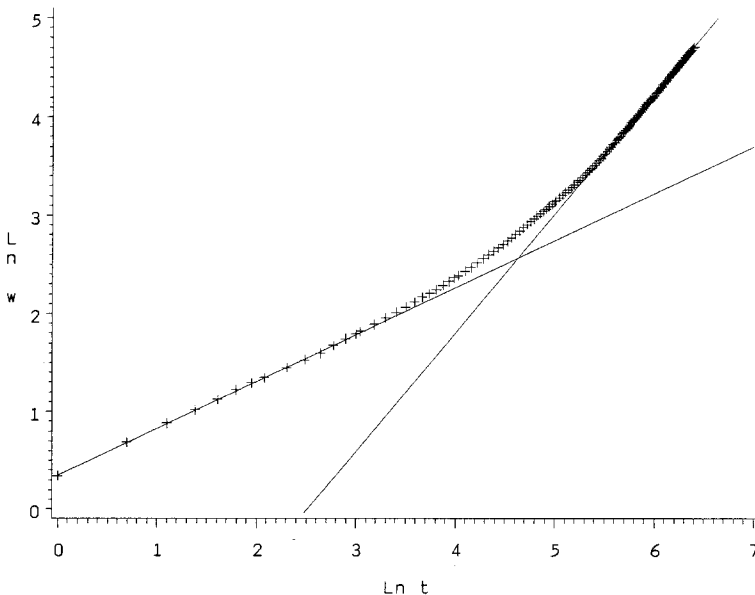


Fig. 5. Log-log plot of w vs. t for $L=1000$; $f=1$. Note two regimes: $\beta \approx 1/2$ and $\beta \approx 1$ (Model 2).

for $f \geq 1$. We suggest that the enhanced algorithm of Model 2 promotes temporal correlations even further, and thus leads to a very complex behavior at short times. However, Model 2 is quasideterministic and may lead to coalescence of mountains. Note that for a deterministic model in which only one mountain is grown, $\omega \sim L$, and thus the result $\beta = 1$ and $\chi = 1$ for $f \geq 1$ is perhaps not too surprising.

4. HEIGHT-HEIGHT CORRELATIONS FOR RANDOM DEPOSITION WITH CORRELATED NOISE

In order to test numerically the conjecture $f = 2\rho$,⁽³⁾ we have used the equation describing random deposition

$$\partial h / \partial t = \eta(x, t) \quad (9)$$

to find a direct relationship between the height-height correlations and the noise correlations. For spatially correlated noise (6) we find

$$\langle \delta h(x, t) \delta h(x', t) \rangle \sim t |x - x'|^{\Delta} \quad (10)$$

where $\delta h = h - \langle h \rangle$; t as usual stands for $\langle h \rangle$ and $\Delta = 2\rho - 1$. Thus, the height-height spatial correlation function provides a way to measure Δ directly from the expression

$$\langle h(x, t) h(x', t) \rangle / \langle h \rangle - \langle h \rangle \sim |x - x'|^{\Delta} \quad (11)$$

calculated for each value of f . We can determine then if indeed $f = \Delta + 1$, as is implied by the conjecture $f = 2\rho$. A typical graph of the expression (11) vs. $|x - x'|$ is shown in Fig. 6. The negative values are perhaps not surprising, since due to periodic boundary conditions, the total area under the curve needs to be zero. However, the curve as a whole does not exhibit a single scaling law. We were able to fit only the first 15 data points, and indeed found that $f \approx 2\rho$ for $f < 1$. Note, however, that the noise-noise correlations (6) are supposed to occur asymptotically for large $|x - x'|$.⁽¹¹⁾ Moreover, $2\rho - 1$ becomes positive for $\rho > 1/2$ and thus we should observe the expression (11) to increase for $f > 1$. We have never witnessed this to occur (see Fig. 6). Thus, the identity $f = 2\rho$ appears doubtful even for unrestricted growth, uncomplicated by the onset of the temporal noise correlations. Any comparison of the discrete correlated deposition models of the types of refs. 1 and 3 to the theoretical prediction⁽¹¹⁾ cannot be made consistently without the understanding of the relationship between f and ρ . One might speculate that even in random correlated deposition the temporal noise correlations are still present, that they are intrinsic to the correlated deposition algorithm itself. The question of connecting the

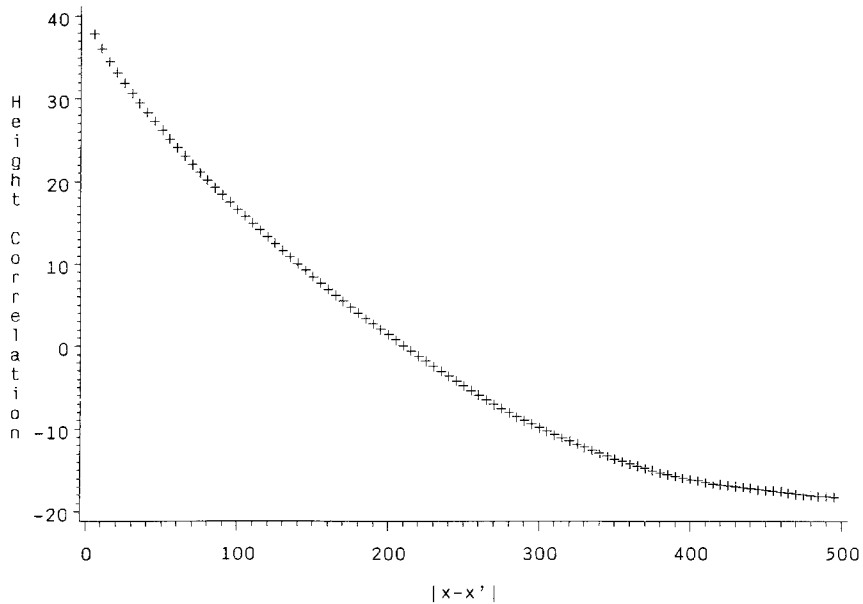


Fig. 6. Plot of $\langle h(x, t)h(x', t') \rangle / \langle h \rangle$ vs. $|x - x'|$ for $f = 2$; $L = 1000$ (random deposition).

height–height correlations in the correlated deposition process to noise correlations thus remains open.

5. CONCLUSIONS

In summary, we have observed a considerable departure in the RSOS model with correlated noise from both theory⁽¹¹⁾ and earlier numerical simulations.⁽³⁾ We have suggested a number of reasons for such a departure and have analyzed the growth behavior and height–height correlation functions for the first time in this context. In conclusion, we conjecture that the temporal noise correlations are inherently present in both our models, thus making the correlated RSOS deposition process rather complex and worth further study. We note that, unlike the fractal dimension of growth perimeters⁽¹⁾ which started to cross over toward the short-range regime at $f = 1$, the surface roughness exponents χ and β seem to conclude this cross-over at $f = 1$. Our results suggest that both exponents approach 1 at $f = 1$, and that the behavior does not change qualitatively for $f > 1$. The height restrictions seem to be overwhelming in this regime. It seems plausible that the influence of an additional operator introduced by such constraints is further promoted and finally made relevant by strongly correlated noise. Finally, we have studied numerically the direct relationship of noise–noise

correlation strength to f , and have found a comparison of discrete growth models^(1,3) to theory⁽¹¹⁾ so far unjustified.

NOTE ADDED IN PROOF

A new theoretical prediction was recently brought to our attention (Y. C. Zhang, private communication, 1990) based on replica method and considered "exact": $\beta = (1 + 2\rho)/(3 + 2\rho)$ for $0 < \rho < 1/2$. We note that if one follows the comparison $f = 2\rho$, our results agree with this new prediction for weakly correlated noise ($\rho < 1/4$). Thus, we may conclude that the time correlations which build up in the process of sequential deposition become relevant only for strong noise correlations. However, we remind that the physical meaning of increasing ρ from 0 (uncorrelated noise) to $1/2$ (limit of long-range correlations) is totally different from that of increasing f from -1 (uncorrelated deposition) to 1 (short-range limit for deposition). As both of these processes seem to lead to increased roughness, we feel that an agreement found may be coincidental.⁽¹⁴⁾ We plan to proceed with a new simultaneous deposition algorithm aiming at the direct comparison with the prediction of Zhang.

ACKNOWLEDGMENTS

We thank M. Kardar, M. Kosterlitz, P. Meakin, and F. Family for many helpful suggestions and for communicating to us their work prior to publication. We appreciate support from Academic Computing at Wellesley College and especially the kind assistance of L. Baldwin. The work of A.M. was supported by a grant from the Du Pont Co.

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